Timer Belts

These have moulded cogs inside cogs inside which acts as flexible internal teeth, by engaging with the axial grooves (teeth) in mating pulleys.

Timer belts can transmit power up to 1000kW at linear speeds from 5-50 m/sec. The transmission ratio can be as high as 12, and the efficiency higher [98%] than in flat belts. Made from oil resistant rubber, time belts can operate even in a pool of oil.

One of the pulleys is provided with a 1.5 – 4 high flange, to prevent axial slip of the belt.

Timer belt pulley P.C.D = $d_p = m_n$.
Where m = module (mm) ; n = No of teeth (grooves)

Transmission ratio should not exceed 6 (or be less than 1/6). Driving pulleys can run without trouble for 100 to 4,000 R.P.M; even up to 5000 R.P.M; for 5 module, and 8000 R.P.M for 3 module.

The pitch ϕ PCD = $d_p = d_o + (H + h)$
Pitch distance of Cogs = $P = \pi m$

The approximate distance can be found from the pitch ϕ (D_p) of the big pulley.

Example
Design timer belts transmission for 5.5kW at 1400 R.P.M driver speed, and 700 driven R.P.M. The system run continuously for 24 hours.

Solution

$$\text{Torque} = \frac{5.5 \times 1000}{1000} = 0.393 \text{ kgm}$$

From Table 12, 5 module belt with a 0.49kgm torque capacity, appears suitable. The minimum number of teeth = 18- 24. Taking the higher value,

$$\text{Driver pulley } \phi = d_p = m_n = 5 \times 25 = 120$$

$$\text{Driven Pulley } \phi = D_p = 120 \text{ x } \frac{1400}{700} = 240$$

From

$$C = \frac{1.5D}{\sqrt[3]{i}} = \frac{1.5 \times 240}{\sqrt[3]{0.5}} = \frac{360}{0.794} = 453.6$$

Also from,

$$C = 0.25\left[L_p - 1.57\left(d + D\right) + \sqrt{\left(L_p - 1.57(d - D)^2 - 2(D - d)^2\right)}\right]$$

$$C = 453.6 = 0.25\,L_p - 1.57\left(d + D\right) +$$

$$\sqrt{\{L_p - 1.57(D + d)\}^2 - 2(D - d)^2}$$

$$= L_p - 565.2 + \sqrt{(L_P^2 - 1130.4L_p + 319451 - 2 \times 120^2}$$

$$= L_p - 565.2 + \sqrt{L_P} - 1130.4L_P + 290651$$

$$2379.6 - L_P = \sqrt{(L_P^2 - 1130.4L_p + 290651}$$

Squaring both sides,

$$5662496 - 4759.2\ L_p + L_P^2 = L_P^2 - 1130.4\,L_P + 290651$$

$$L_p = \frac{5662496 - 290651}{4759.2 - 1130.4} = 1480.33$$

No of cogs $= \frac{L_P}{P} = \frac{1480.33}{15.71} = 94.22 \cong 94$ (integer)

: Belt Pitch length = 94 × 15.71

= 1476.74

The belt is usually specified by the number of pitches (cogs) ie 94 pitches of 5 module in this case.

Chain Drive

Instead of belt, a more compact and positive roller chain and sprockets if the linear speed is less than 12m/sec (at the most 20m/sec) and transmission ratio is less than 7. The flexibility of the chain makes the drive shock absorbent. The number of teeth on the smaller sprocket should not be less than 17 (Preferably 21). The number of teeth on the other sprocket can be determined from

$$\text{i} = \frac{N_n}{N_l} = \frac{No\ of\ teeth\ on\ bigger\ sprocket}{No\ of\ teeth\ on\ smaller\ sprocket} = \frac{t_n}{t_l}$$

C_{min} = 20p or 1.5D; C_{max} = 80p

P = Chain roller pitch; D = Bigger Sprocket ϕ.

Chain length = L = $2\frac{C}{P} + \frac{t_1 - t_2}{2} + (\frac{t_2 + t_1}{2\pi})^2 \frac{P}{C}$

where

t_1 , t_2 = No of teeth on sprockets; C = Centre distance p = chain pitch.

L should be an integer multiple of the chain pitch (preferably even), to avoid a special (cranked) link.

Pitch ϕ of sprocket (d or D) = $\frac{P}{sin\frac{180}{t}}$

For smaller sprocket with 17 teeth

$$d = \frac{P}{sin\frac{180}{17}} = 5.4422p$$

Example

Design a roller chain drive for transmitting 150 kW from a 1370 R.P.M. motor to obtain 400 R.P.M. at the output spindle that is subjected to a highly pulsating load.

Solution

From $d = \frac{P}{sin\frac{180}{t}}$, d = 5.4422p

For smallest 05 B chain with 8 pitch

$$d = 5.4422 \times 8 = 43.53$$

Torque = T = $\frac{150\ X\ 100}{1370} = 10.95$kgm

= 10950 kgm

$F = \frac{10950}{43.53/2} = 503.\ 05$ kg, i.e. much more than the breaking load 460 kg, of a single (simplex) chain.

Although duplex chain with 800kg breaking load will suffice, it will be safer to use a triplex chain with 1140 kg breaking load. Then, factor of safety will be

$$= \frac{1140}{503} = 2.266$$

Alternatively, 06 B chain with 9.525 pitch can be use

Then d = 5.4422p = 5.4422 x 9.525

= 51.84

F = $\frac{10950}{51.84/2}$ = 422.45 ie much lesser than the breaking load 910 kg of a single simplex chain.

For the driven sprocket teeth = i = $\frac{t_{out}}{t_{in}}$

i = $\frac{N_n}{N_l} = \frac{1370}{400}$ = 3.435

$t_{out} = t_{in}$ x i = 17 x 3.425 = 58.225 ≅ 58 (integer)

Min. center distance = 20p or 1.5D

20p = 20 x 9.525 = 190.5

$D\frac{P}{sin\frac{180}{t}} = \frac{P}{sin\frac{180}{58}}$ = 175.93

:. 1.5D = 1.5 x 175.93 = 263.9

Selecting the higher value of 263.9

Chain length (L) $= 2\frac{C}{P} + \frac{t_1 - t_2}{2} + (\frac{t_2+t_1}{2\pi})^2\frac{P}{C}$

$= 2\frac{263.9}{9.525} + \frac{17-58}{2} + (\frac{58-17}{2\pi})^2\frac{9.525}{263.9}$

= 55.41 + 37.5 + 1.537

= 94.446 ≅ 95

This can be evened to 96 pitches, avoid a cranked link. Then center distance ‘ C’ will be suitable for a highly pulsating load. The overall length of the chain will be 96 x 9.525 = 914.4mm

GEAR DESIGN

Module is selected to suit the bending load.

For straight spur gears:

$$m = 126 \sqrt[3]{\frac{T}{Y \ f_b \Psi_m t_1}}$$

$$T = Torque \ (km \ cm)$$

Y = Form factor (from Table 13)

F_b = Permissible bending stress (kg/cm^2) (From Table 14)

ψ_m = which factor = $\frac{b}{m}$ (From Table 15)

t_1 = No. of teeth of pinion (smaller gear)

For Helical Gears:

$$m = 11.5 \cos \beta \sqrt[3]{\frac{T}{Y_e \ f_b \ \psi_m \ t_1}}$$

Y_e = from factor based on equivalent

No. of teeth t_v

$t_v = \frac{t}{\cos^3 \beta}$; β = Helix angle, generally 8^o-45^o

The module can also be found from the graph in [Fig. 5] . The intersection of the (doted) horizontal line for 1,313kg cms torque, intersects the curve for, f_b=1,400 kg/cm^2, at a point which gives module 3.6 = 3.5 (Std).

For straight spur gears, $f_b = \frac{10(i \pm 1)T}{Cmb \ Y}$

For Helical spur gears, $f_b = \frac{7(i \pm 1)T}{Cmb_n Y_v}$

Note in both these equation $i \geq 1$. For in gears pair,

$i = \frac{No. \ of \ teeth \ of \ (bigger) gear}{No. of \ teeth \ of \ (smaller) \ pinion}$; the sign + for external gears

- for internal gears

The measure of the wear resistance, the compressive or surface stress can be found from

$$f_c = 0.74 \left\{\frac{i \pm 1}{C}\right\} \sqrt{\frac{i \pm 1}{ib} ET}$$

Note that E = Elasticity modulus for pinion and gear materials combination $= \frac{2E_1E_2}{E_1+E_2}$

The value of the surface stress should be less than the permissible (Table 14.) limits.

For helical spur gears,

$$f_{ch} = 0.7 \left\{\frac{i \pm 1}{C}\right\} \sqrt{\frac{i \pm 1}{ib} ET} \qquad \text{The sign + for external gears}$$

Example 1

Design a pair of gears for speed-up (or step-up) transmission ratio (i) = 8, power to be transmitted = 1.1kW. Driver R.P.M. range 80-500 Revs/min

Solution: Power = 1.1kW; At the slowest level of 80 R.P.M., the torque will be the highest

$$\therefore\ T = \frac{1.1 \times 955.0}{80} = 13.13\ kgm$$

$$= 1313 kgcm$$

From Table 15 for unground gears, $\psi_m \frac{b}{m} = 6$

From table 14 for C40

$$f_c = 45 kg/mm^2 = 4{,}500\ kg/cm^2$$

If we take a pinion having 20 teeth, the gear will have 20 x 8 = 160 teeth. From Table 13, form factor for a standard 20 teeth pinion Y_{20}= 0.389, and for 160 teeth Y_{160} = 0.516

From eqn.

$$m = 12.6 \sqrt[3]{\frac{T}{Y\ f_b\ \psi_m\ t_1}}$$

$$= 12.6 \sqrt[3]{\frac{1313}{0.389 \times 1400 \times 6\ \times 20}} = 3.4$$

~ 3.5

The module can also be found from the graph in [Fig. 5]. The (dotted) horizontal line for 1,300 kg/cm torque intersects the curve for 1,400 kg/cm^2, bending stress at the point, which gives 3.6mm as the module. It can also be noted that the intersection of the torque line with the 500kg/cm^2 stress curve gives 5 mm module.

For a gears having 20 and 160 teeth, standard center distance = C

$$C = \frac{m(t_1 + t_2)}{20} = \frac{3.5(20 + 160)}{20} = 31.5 \ cm$$

$$b = \psi_m \times m = 6 \times 3.5 = 21mm = 2.1 \ cm$$

For external straight spur gears, the sign for bracketed terms is +

$$f_b = \frac{10(i+1)T}{CmbY} = \frac{10\ (8+1)1313}{31.5 \times 3.5 \times 2.1 \times 0.389}$$

$$= \frac{1,18,170}{90.06}$$

$$= 1,312$$

Let us check up the gear for compressive stress

For the elastic modulus for the combination of steel gear and pinion

$$E = \frac{2E_1E_2}{E_1 + E_2} = \frac{2 \times 2.15 \times 10^6 \times 1.1 \times 10^6}{2.15 \times 10^6 + 1.1 \times 10^6} = 1.46 \times 10^6$$

From

$$f_c = 0.74 \left\{\frac{i \pm 1}{C}\right\} \sqrt{\frac{i \pm 1}{ib} ET}$$

$$= 0.74 \left(\frac{8+1}{31.5}\right) \sqrt{\frac{8+1}{8 \times 2.1} \times 1.46 \times 10^6 \times 1313}$$

$$= 6775.5 \ i.e. > 4500 \ \ for \ C40 \ (Table \ \ 14)$$

We can find the center distance (C) suitable for obtaining 4500/cm^2 compressive stress from the above equation. Using the present gear width center distance ratio, $\psi_c = \frac{2.1}{31.5} = 0.0666$

$$f_c = 4500 = 0.74 \left(\frac{i+1}{C}\right) \sqrt{\frac{i+1}{ib} ET}$$

$$= \frac{0.74 \times 9}{C} \sqrt{\frac{9}{8 \times 0.0666C} \times 1.46 \times 10^6 \times 1313}$$

$$\therefore \quad C = 41.4 \ cms$$

$$= \frac{m}{20}(20+160)$$

$\therefore\ m = 5.3 = 5.5$ the next higher std module. The module can also be found from the graph [Fig 5b] (dotted line). It is 4.05, but for b = 0.15C.

Example 2.

Design a pair of gears for transmitting 11 kW at 1:2 speedup, at driver R.P.M.s ranging from 200-630. Use steel for both gears. Desired Life = 10^7 cycles

Solution

power = 11 kW = 11,000 Watts/sec.

Min. R.P.M. = 200 for driver

$$\text{Max torque } (T) = \frac{11\times 95{,}500}{200}$$

$$=5242.5 \text{ kgcm}$$

We will design the gears for wear-load first. From Table 14 , let us choose C 40 (175 BHN) for wheel and C45 (230 BHN) for the pinion.

For 175 BHN steel and 10^7 cycles life permissible,

$f_b = 1400\,kg/cm^2$, *and* $f_c = 4500\ kg/cm^2$.

For 230 BHN steel with same life, $f_b = 1800$ *and* $f_c = 5000$. The pinion hardens should be at least 50 BHN higher than the wheel hardness, to balance the war of the two gears. Higher R.P.M, wears out the pinion more than the lower R.P.M. gear

From Eqn. for external gears:

$$f_c = 0.74\left(\frac{i+1}{C}\right)\sqrt{\frac{i+1}{ib}ET}$$

For steel to steel combination $E = 2.15\times 10^6/cm^2$

Width (b) = 0.15C for medium speeds (Table 3)

$$f_c = 0.74\left(\frac{2+1}{C}\right)\sqrt{\frac{2+1}{2\times 0.15C}\times 2.15\times 10^6 \times 5{,}252.5}$$

$$= \frac{746028}{C^{1.5}}$$

$$Taking\ f_c = 4{,}500,\ C^{1.5} = \frac{746028}{4{,}500}$$

$$C = 30.18/cm = \frac{m(t_1+t_2)}{20}$$

$$Taking\ t_2 = 40\ and\ t_1 = 20\ for\ i = 2$$

$$\frac{m(20+40)}{20} = 30.18$$

$$m = \frac{30.18\times 20}{60} = 10.06 = 10,\ i.e.\ very\ high$$

$$b = 0.15C = 0.15\times 30.18 = 4.53\ cms = 45mm$$

The gears to be used must be ground. Therefore, the unhardened, unground and unlapped gears of the 9th accuracy class cannot be used.

The gear sizes can be reduced by using considerably hardened gears. Referring to Table 14, it can be observed that case hardening steels can allow the usage of a much higher (9,500 kg/cm^2) value for compressive stress (f_c) *For* $f_c = 9{,}500$

$$C = \sqrt[1.5]{\frac{746028}{9{,}500}} = 18.34$$

$$Module = \frac{18.34\times 20}{60}\quad 6.11 = 6.5$$

We can use 13Ni 3Cr80 with 255 BHN for the gear, and 15Ni2Cr1Mo15 with 330 BHN for the pinion, to provide the required difference in hardness (Min.50 BHN) between the pinion and the gear.

Contact ratio (E)

For minimizing noise and impact, the meshing gears must always be in contact. The following pair of teeth should engage before the preceding pair separates. Contact ratio (E) is a measure of such contact. It can be measured along the profile (E_α) or along the face (E_β). The latter being of little significance, is disregarded.

$$E_\alpha = \frac{Angle\ of\ contact\ of\ one\ tooth}{Angular\ pitch,\ i.e. \frac{2\pi}{t}}$$

For straight spur gears with t_1 and t_2 teeth

$$E_\alpha = 1.88 - 3.2\left(\frac{1}{t_1} \pm \frac{1}{t_2}\right)$$

Note: The minus sign is sued for internal gears. Contact ratio E_α must be equal to or more than 12.

Example 3.

Find the profile contact ratio for pairs of straight spur gears with a standard profile (with no correction) and no of teeth.

(a) 20 and 160
(b) 20 and 40

Solution

(a) $E_\alpha = 1.88 - 3.2\left(\frac{1}{20} + \frac{1}{40}\right) = 1.64$
(b) $E_\alpha = 1.88 - 3.2\left(\frac{1}{20} + \frac{1}{160}\right) = 1.67$

For helical gears, with helix angle (β):

$$E_{\alpha h} = \frac{E_\alpha}{\cos\beta} = \frac{1.88 - 3.2\left(\frac{1}{t_1} + \frac{1}{t_2}\right)}{\cos\beta}$$

Note: The minus sign is used for internal gears.

As Cos β will always be less than one, it is obvious that an increase in the helix angle will increase the contact ratio up to 45^o.

Helical gears

Slanting teeth with a rotation axis, facilitates gradual engagement. The module in the plane normal to the tooth flanks is called the normal module (m_n). The inclination of the tooth with the axis changes the module in the plane square to the axis. This transverse module (m_t) determines the pitch diameter (d_p), the transverse pressure angle (α_t), and lead (L). The angle with the axis (β), the helix angle, determines the transverse and overall dimensions.

Transverse module $m_t = m_n\left(\frac{1}{\cos\beta}\right)$

Pitch $\phi(d_p) = tm_t = tm_n\left(\frac{1}{\cos\beta}\right)$

Base circle $\phi(d_p) = d_p \cos\alpha_t$

$$\tan \alpha_t = \tan\ \alpha_n \times \left(\frac{1}{\cos \beta}\right)$$

$$\alpha_t \ = \text{Transverse pressure angle}$$

Virtual no. of teeth = $t_v \ = \dfrac{t}{\cos^3\ \beta}$

Center distance also changes with the pitch diameters

$$C = \frac{d_p\ +\ D_p}{20}\ = \frac{(t_1 + t_2)}{20 \cos\ \beta}\ cms$$

$$Lead = L\ = \pi d_v\ \cot\ \beta$$

Note: C is in cm while d_p, D_p and L are in mm

It should be remembered that the helix angle does not affect addendum or dedendum, which correspond with the normal module (m_n), the module of the cutting tool: milling, shaping or hobbing cutters.

Example 4

Find the important dimensions for a 40 teeth, 2 module gear, with a 10^o helix angle. The width (b) can be taken as 7 times the module. Normal pressure angle (α_n) = 20^o

Solution:

From $m_t \ = m_n \left(\dfrac{1}{\cos \beta}\right)$

$$m_t\ = m_n\ \frac{1}{\cos\ \beta}\ = 2\ \frac{1}{\cos\ 10^o}\ = 2.0308$$

$$d_p = t\ m_n\ .\ \frac{1}{\cos\ \beta}\ = 40 \times 2 \times \frac{1}{\cos 10^o}$$

$$= 81.23$$

$$Addendum = m_n\ = 2$$

$$Dedendum\ = 1.25\ m_n\ = 2.5$$

$$Outside\ \phi = d_o\ = 81.23 + 2 \times 2 = 85.23$$

$$Root\ \phi = d_t\ = 81.23 - 2 \times 2.5 = 76.23$$

Transverse pressure angle = α_t

$$\alpha_t = \tan^{-1}\left(\tan\,\alpha_n\,\frac{1}{\cos\,\beta}\right) = \tan^{-1}\left(\tan\,20.\,\frac{1}{\cos 10^o}\right)$$

$$= 20.283^o = 20^o\ 17'$$

$$\text{Virtual no. of teeth} = t_v = \frac{40}{\cos^3 10^o} = 41.88$$

$$= 42$$

$$Lead = L = \pi\, d_p\, \cot\,\beta$$

$$= \pi \times 81.23\,.\ \cot 10^o$$

$$= 1447.84\ mm$$

Designing gears for given center distance

When space constraints call for a change in the center distance, addendum modifications can be used to accomplish the same.

The following parameters define such changes:

1. Working (required) center distance C^1 cm
2. Average no. of teeth, $'t_m' = \frac{t_1 + t_2}{2}$
3. Center distance modification coefficient = K_c

$$K_C = \frac{(C - C)10}{m}$$

$$C = Std.center\ dis\tan ce = \frac{m(t_1 + t_2)}{20} cm$$

4. Working pressure angle = α'

   $$\text{Cos}\ \alpha' = \frac{Cos\ \alpha}{C'}$$

5. Sum of modification coefficients $(x_1 + x_2)$ for pinion (x_1) $and\ gear$ (x_2). The value of the sum can be found from Table 16.

6. Addendum reduction coefficient = K_{ar}

   $K_{ar} = x_1 + x_2 - K_c$

7. Addendum = $h_a = (1 + x - K_{ar})$ m,

   Where $x = \frac{x_1+x_2}{2}$

8. Dedendum = $h_d = (1.25 - x)\ m$
9. Outside $\phi = d_0 = t + 2 + 2x - 2K_{ar}$
10. Root $\phi = d_r = (t - 2(1.25 - x))m_n$

11. Contact ratio (E) is the angle of action of one of the gears, divided by its angular pitch. A decrease in the center distance, decreases the pressure angle [α] and contact ratio.

$$E = \frac{1}{2\pi}\left(t_1\left(\tan\theta_1 - \tan\alpha'\right) + t_2\left(\tan\theta_2 - \tan\alpha'\right)\right)$$

$\theta_1 = cos^{-1}\left\{\frac{d_b}{d_o}\right\}$; $\theta_2 = cos^{-1}\left\{\frac{D_b}{D_o}\right\}$ D_b = Base circle dia. of bigger gear

D_O = Outside dia. of bigger gear

Contact ratio (E) for helical spur gears is calculated in the transverse plane, with transverse module (m_t) and mean virtual number of teeth (t_{mv}).

$$t_{mv} = \frac{t_{v1} + t_{v2}}{2}$$

$$\cos\alpha' = \frac{C}{C'}\cos\alpha$$

$$\text{As } C = \frac{d_p + D_p}{20}$$

$$\frac{x_1+x_2}{t_m} = \frac{inv\,\alpha' - inv\alpha}{\tan\alpha}, \quad \frac{K_c}{t_m} = \frac{\cos\alpha}{\cos\alpha'} - 1$$

$$\cos\alpha'_t, \quad \frac{d_b + D_b}{20C'}$$

Note: C'in cms, while d_b and D_b are in mm

$$\frac{K_C}{t_m'} = \frac{1}{\cos\beta}\left\{\frac{\cos\alpha_t}{\cos\alpha_t} - 1\right\}$$

Example 5

Design gears for 6.8 cm center distance and 1:3 d step-up transmission. $\alpha = 20^o$

Solution: For 1:3 ratio $t_1 = 3t_2$

$$\frac{(t_1 + t_2)}{20} = \frac{(3t_1 + t_2)m}{20} = \frac{4t_2 m}{20}$$

If we take $t_2 = 20$; $\frac{4 \times 20 \times m}{20} = 6.8$

$$\therefore\ m = 1.7$$

If we take m = 1.75,

$$C = \frac{4 \times 20 \times 1.75}{20} = 7cms$$

$$For \quad C' = 6.8$$

$$From\ eqn\ 2.50\ \ K_C = \frac{(6.8 - 7)10}{1.75} = -1.1428$$

$$t_m = \frac{60 + 20}{2} = 40$$

$$\alpha' = \cos^{-1}\left\{\frac{C}{C}\cos\alpha\right\} = \cos^{-1}\left\{\frac{7}{6.8}\cos 20^o\right\}$$

$$= 14.6858 = 14^o\ \ 41.15'$$

$$x_1 + x_2 - t_m \left\{\frac{inv\alpha' - inv\ \alpha}{\tan\alpha}\right\} - 10\left\{\frac{inv 14^o 41' - inv\ 20^o}{\tan 20^o}\right\}$$

$$= 40\left(\frac{0.0057647 - 0.014904}{0.36397}\right) = 1.0044$$

$$x = \frac{-1.0044}{2} = 0.5022$$

Referring to mark point for $t_m = 40$ and x = -0.5.

Draw a dotted line in the mean position (Refer to Fig. 4). The intersection points of this line with the vertical lines for 20 and 60 teeth, give pinion correction coefficient (x_1) and gear correction coefficient (x_2), as -0.33 and -0.57, respectively.

$$K_{ar} = x_1 + x_2 - K_C = -1.0044 - (-1.1428)$$
$$= 0.1384$$
$$h_{ap} = m(1 + x_1 - K_{ar}) = 1.75 \ (1 + (-0.33) - 0.1384)$$
$$= 0.93103$$
$$d_p = mt_2 = 1.75 \times 20 = 35$$
$$d_v = 35 + (2 \times 0.9303) = 36.86$$
$$d_b = d_p \cos\alpha' = 35 \cos 14.686^o = 33.856$$
$$D_p = mt_1 = 1.75 \times 60 = 105$$
$$D_b = 105 \cos 14.686^o = 101.57$$
$$h_{ag} = m(1 + x_2 - K_{ar}) = 1.75(1 - 0.57 - 0.1384)$$
$$= 0.9303$$
$$D_v = 105 + (2 \times 0.5103) = 106.021$$
$$\theta_1 = \cos^{-1}\left(\frac{d_b}{d_v}\right) = \cos^{-1}\left(\frac{33.856}{36.86}\right) = 23.29^o$$
$$\theta_2 = \cos^{-1}\left(\frac{D_b}{D_v}\right) = \cos^{-1}\left(\frac{101.57}{106.021}\right) = 16.661^o$$
$$E = \frac{1}{2\pi}\left(t_1(\tan\theta_1 - \tan\alpha') + t_2(\tan\theta_2 - \tan\alpha')\right)$$
$$= \frac{1}{2\pi}\left(20(4302 - 0.2621) + 60\ (0.0373)\right)$$
$$E = 0.89 \le 1.2, \ \textit{and therefore, very low.}$$

The contact ratio can be increased by increasing the number of teeth. This will entail reducing the module. If we change the module to 1.25, t hen,

$$C = 6.8 = \frac{4t_2 m}{20} = 6.8 = \frac{4 \times t_2 \times 1.25}{20}$$
$$t_1 = 27.2 = 27 \ \textit{an} \text{ int}\textit{eger}$$
$$t_2 = 3 \times 27 = 81$$
$$C = \frac{m(t_1 + t_2)}{20} = \frac{1.25(81 + 27)}{20} = 6.75 cms$$
$$\alpha' = \cos^{-1}\left(\frac{C}{C}\cos\alpha\right) = \cos^{-1}\left(\frac{6.75}{6.8}\cos 20^o\right) = 21.127^o = 21^o 7.63'$$
$$t_m = \frac{81 + 27}{2} = 54$$

Referring to Table 16, for 21.127° = 21°7.63', $\frac{x_1 + x_2}{t_m}$ = 0.0076

$$x_1 + x_2 = 54 \times (+0.0076) = 0.4104$$

$$x = \frac{x_1 + x_2}{2} = 0.20521$$

Refer to Fig. 6, Mark point for 54 teeth and +0.2 correction. The correction line should be between $i = \frac{1}{7}\, and \frac{1}{6}$. The intersection points of the vertical lines, for 27 teeth and 81 teeth with the line, give the pinion and gear correction as: x_1 = + 0.16 and x_2 = +0.23, respectively.

From the table 16 for 21°7.63'; $\frac{K_c}{t_m} = +0.007412$

$$K_c = 54(+0.00741) = 0.4002$$

$$K_{ar} = x_1 + x_2 - K_c = (+0.4104) - (0.4002)$$

$$= 0.010152$$

$$h_{ap} = 1.25(1 + (0.16) - 0.01015) = 1.437$$

$$d_v = (1.25 \times 27) + (2 \times 1.437) = 36.624$$

$$d_b = d_p \ \cos\alpha' = 33.75 \cos 21.127^o = 31.48$$

$$\partial_1 = \cos^{-1}\left(\frac{d_p}{d_v}\right) = \cos^{-1}\left(\frac{31.48}{36.624}\right) = 30.121^o$$

$$D_v = (1.25 \times 81) + 2(1 + 0.23 - 0.01015)1.25$$

$$= 104.30$$

$$D_b = D_p \ \cos 21.127^o = 94.444$$

$$\theta_2 = \cos^{-1}\left(\frac{D_b}{D_v}\right) = \cos^{-1}\left(\frac{94.444}{104.3}\right) = 25.108$$

$$E = \frac{1}{2\pi}(20(\tan 20.121 - \tan 21.127) + 60\ (\tan 25.108 - \tan 21.127))$$

$$= \frac{1}{2\pi}(3.875 + 4.931) = 1.401$$

The contact ratio is more than the recommended minimum of 1.2.

Example 6

Find out the necessary tooth modifications if 20 teeth and 40 teeth, 1.75 module reduction gears have to operate at a center distance of 54, instead of the standard center distance of 52.5. Teeth pressure angle (α) = 20°.

Solution: Center distance modification coefficient (K_v) should be found first.

From

$$K_C = \left(\frac{C'-C}{m}\right) = \frac{54-52.5}{1.75}$$
$$= 0.857$$

From

Pressure angle $\alpha' = \cos^{-1}\left(\frac{C}{C'}\cos\alpha\right) = Cos^{-1}\left(\frac{52.5}{54}\cos 20^o\right)$

$$\alpha' = 24^o$$

From table 2.27 for $\alpha' = 24^o$

$$\frac{x_1 + x_2}{t_m} = 0.03145;\ and\ \frac{K_c}{t_m} = 0.02862$$
$$t_m = \frac{20 + 40}{2} = 30$$
$$\frac{x_1 + x_2}{30} = 0.03145;\ Thus\ x_1 + x_2 = 30\times 0.03145 = 0.9435$$
$$d_p = d_p \cos 24^o = 20 \times 1.75 \times 0.913$$
$$= 31.974$$

$$D_b = D_p \cos 24^o = 40\times 1.75 \times 0.913$$
$$= 63.91$$
$$\therefore\ x_1 + x_2 = 0.9435$$
$$x = \frac{0.9435}{2} = 0.4717$$
$$\frac{KC}{30} = 0.02862$$
$$K_C = 0.8586$$

The approximately matches the value 0.857, found earlier.

Refer to (Fig. 7). Following the vertical line for t_m, i.e. 30 teeth, we market he intersection point for x = 0.47 and draw a dotted line, as shown in the mean position. The intersection of the dotted line with the line for 20 teeth gives us x_1=0.47 for the pinion. Similarly, for a 40 teeth gear, x_2 =

0.45.

$$\therefore\ K_{ar} = x_1 + x_2 - K_C = 0.9435 - 0.858$$
$$= 0.0855$$

Pinion addendum $[h_a] = (1 - x_1 - K_{ar})\, m = (1 + 0.47 - 0.0855)1.75$

$$= 2.423$$
$$d_v = 20 \times 1.75 + 2 \times 2.423$$
$$= 39.846$$
$$\theta_1 = \cos^{-1}\left(\frac{d_b}{d_v}\right) = \cos^{-1}\left(\frac{31.974}{39.846}\right) = 36.636^o$$

Gear addendum = $(1 + x_2 - K_{ar})\, m = (1 + 0.45 - 0.0855)1.75 = 2.38$

$$D_v = 40 \times 1.75 + 2 \times 2.36$$
$$= 74.776$$
$$\theta_2 = \cos^{-1}\left(\frac{D_b}{D_v}\right) = \cos^{-1}\left(\frac{63.91}{74.776}\right)$$
$$= 31.275^o$$

$$E = \frac{1}{2\pi}(t_1 \tan\theta_1 - \tan\alpha') + t_2(\tan\theta_2 - \tan\alpha')$$
$$= \frac{1}{2\pi}\left(20\left(\tan 36.636^o - \tan 24^o\right) + 40\left(\tan 31.275^o - 0.4452\right)\right)$$
$$= \frac{1}{2\pi}(5.968 + 6.487)$$

=1.98, i.e. more than 1.2, which is necessary for a smooth, impact-free transmission.

It can be concluded from the preceding two examples that,

1. When the center distance is reduced below the standard (C' < C), the addendum of the gears is described, while the dedendum is increased. Reduction in the center distance, reduces the pressure angle, and the contact ratio.
2. When the center distance is increased above standard (C" > C'), the addendum of the gears is increased, while the dedendum is decreased. Increased in center distance also increases the pressure angle, and the contact ratio.

Increase in addendum thins the tooth and decreases the tip thickness. The tip thickness should not be less than 0.3m. Referring to the Fig.8 , it can be noted that the point for 0.47 correction and 20 teeth, lies well below the lien for 0.3m tip thickness.